Determining β^* in the Tevatron

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Abstract

Using the two additional Beam Position Monitors (BPM's) found on either side of one of the Interaction Points (the so-called Collision Point Monitors), one can determine, in principle, the single-turn matrix for one BPM location or the other and, hence, the lattice functions at that location. Once the amplitude function and its slope at one BPM is found, it is straight forward to compute the amplitude function through the collision hall and the location of its minimum, and the value of the function at the collision point (β^*).

The two Beam Position Monitors (BPM's), or Collision Point Monitors, within an Interaction Region are located about ± 7.5 m from the Interaction Point (IP) with no intervening magnetic elements (with the exception of the detector solenoid, the effects of which we will ignore for now). Thus, the slope of the trajectory can be accurately determined from the two BPM readings. If one could gather position data at the two BPM's on a turn-by-turn basis, then the inferred slope, coupled with the data at one BPM, can be used to reconstruct the phase space evolution of an induced betatron oscillation over a number of turns, as was first performed in the Tevatron in 1985.[1] For more accurate results, one must appropriately take into account the decoherence of the kicked beam. (See, for example, [2], [3].)

The resulting phase space ellipse can be fit for the local value of the amplitude function and its slope. Once the amplitude function and its slope are determined at one of the BPM's, then it is straight forward to determine the amplitude function at the IP, or any point in between, since the amplitude function is a parabola through this region. Since the BPM's at these locations measure both horizontal and vertical motion, the full 4×4 matrix can in principle be measured and the full effects of local coupling could be inferred. We will defer further discussion of coupled motion for now.

Suppose a betatron oscillation is induced by a fast kicker in the Tevatron. Let x_1 and x_2 be the recorded positions of the oscillation as measured at the upstream and downstream BPM's, respectively, across an Interaction Region straight section, as indicated in Figure 1. The two BPM's are located $\pm L^*$ away from the IP. Thus, the slope of the trajectory at the first BPM is given by

$$x_1' = \frac{x_2 - x_1}{2L^*}.$$

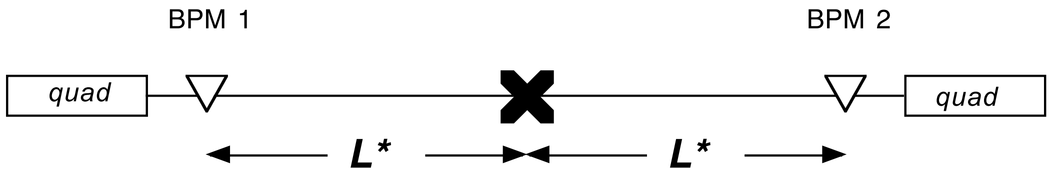


Figure 1: **Schematic of BPM placement across straight section.**

The phase space ellipse of the trajectory at the location of the first BPM is given by

$$\frac{1+\alpha_1^2}{\beta_1}x_1^2 + 2\alpha_1 x_1 x_1' + \beta_1 {x_1'}^2 = A^2.$$

By inducing betatron oscillations with a kicker magnet and taking turn-by-turn BPM data, one can fit the data for A, β_1, and α_1. While different kicker settings will yield different values of A, the data sets should converge toward a consistent set of values for β_1 and α_1. The issue will be the requirements on BPM accuracy versus kick amplitude to make more precise measurements of the amplitude function.

Once the conditions at the upstream BPM are determined, the amplitude function across the straight section will be given by

$$\beta(z) = \beta_1 - 2\alpha_1(z + L^*) + \gamma_1(z + L^*)^2$$

where $\gamma \equiv (1+\alpha^2)/\beta$ and z is the distance from the IP (i.e., $z = 0$ at the center of the detector).

To illustrate the procedure, the pages attached below show the result of a Mathcad simulation of how one would perform the measurement. In the calculation we generate simulated turn-by-turn BPM data at the two detectors, with a random measurement error. In this example, the amplitude measured at the downstream BPM is chosen to be 5% larger than the amplitude measured at the upstream BPM (here, 2 mm) and a random error with rms 20 μm is assigned to each BPM measurement. The slope on each turn between the two detectors is computed, and the data are fit to minimize the function

$$\mathcal{M} = \sum_i \left\{(\alpha_1 x_i + \beta_1 x_i')^2 - a^2\right\}$$

using the three parameters β_1, α_1, and a. The results of the fit are shown, and the resulting function $\beta(z)$ between the two BPM's is plotted. Also shown are the values of β^* (β at $z = 0$), β_{min} and the location (in z) of β_{min}.

Notes on further analysis

Further work needs to be done to understand the applicability of the technique:

- We are most interested in measuring β^* at collision, where the amplitude of the kick should necessarily be small. Thus, the interplay of kick amplitude and BPM resolution must be addressed.
- Similarly, the inherent nonlinearities of the Tevatron will cause the initially coherent betatron oscillation induced by a kicker magnet to decohere, and so the relevant time scales of the decoherence and our ability to extract the true oscillation amplitude and phase space development must be understood.

  That is, we need to determine the required kick amplitude, BPM resolution, and decoherence time necessary to arrive at, say, a 5% measurement of β and α at the BPM.
- It would also be good to look into the applicability of an "AC dipole," as has been used at BNL, for generating sustained driven betatron oscillations for diagnostic purposes.[4],[5] This may be very useful not only for use at the IR's with regards to the above measurement, but also for use with the new high-resolution Tevatron BPM system forthcoming for general "non-destructive" lattice measurements.

References

[1] D. A. Edwards, R. P. Johnson, and F. Willeke, "Tests of Orbital Dynamics using the Tevatron," *Part. Accel.*, **19** 145 (1986).

[2] A. Chao, *et al.*, "Experimental Investigation of Nonlinear Dynamics in the Fermilab Tevatron," *Phys. Rev. Lett.* **61** 2752 (1988).

[3] R. Meller, *et al.*, "Decoherence of a Kicked Beam," SSC Report SSC-N-360, May 29, 1987.

[4] M. Bai, *et al.*, "Experimental test of coherent betatron resonance excitations," *Phys. Rev.* **E56** 6002 (1997).

[5] M. Bai, *et al.*, "Overcoming intrinsic spin resonances with an rf dipole," *Phys. Rev. Lett.* **80** 4673 (1998).

Turn-by-turn IP BPM data to determine Beta-star... MJS 2/5/04

Assume N_{turn}'s of BPM data at the two BPMs across a IP straight section:

$N_{turn} := 200 \qquad i := 0, 1 .. N_{turn}$

$\phi_0 := \mathrm{rnd}(2\cdot\pi)$

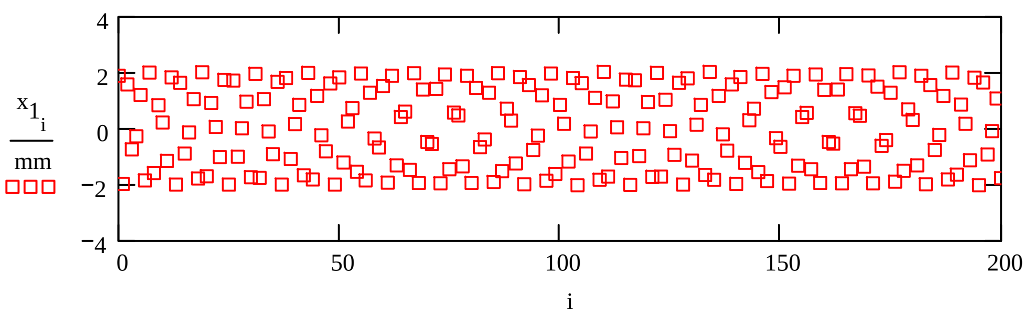
Typical Amplitude: $a := 2\cdot\mathrm{mm}$

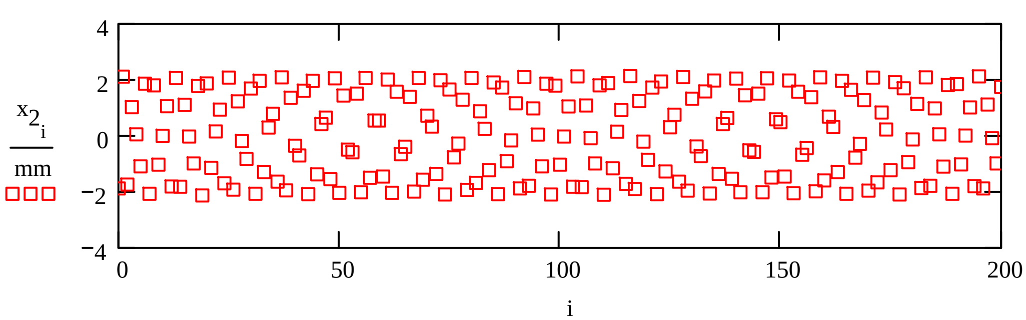
Resolution (mm): $res := 0.020\cdot\mathrm{mm}$

$L_{star} := 7.5\cdot\mathrm{m} \qquad \nu := 0.582408625$

$$x_{1_i} := a\cdot\cos\left(2\cdot\pi\cdot\nu\cdot i + \phi_0\right) + (res)\cdot\mathrm{rnorm}\left(N_{turn} + 1, 0, 1\right)_i$$

$$x_{2_i} := (1.05\cdot a)\cdot\cos\left[2\cdot\pi\cdot\nu\cdot i + \left(\phi_0 + 174.59882873\cdot\mathrm{deg}\right)\right] + (res)\cdot\mathrm{rnorm}\left(N_{turn} + 1, 0, 1\right)_i$$





Determine angles across the straight section: $xp_1 := \dfrac{x_2 - x_1}{2 \cdot L_{star}}$

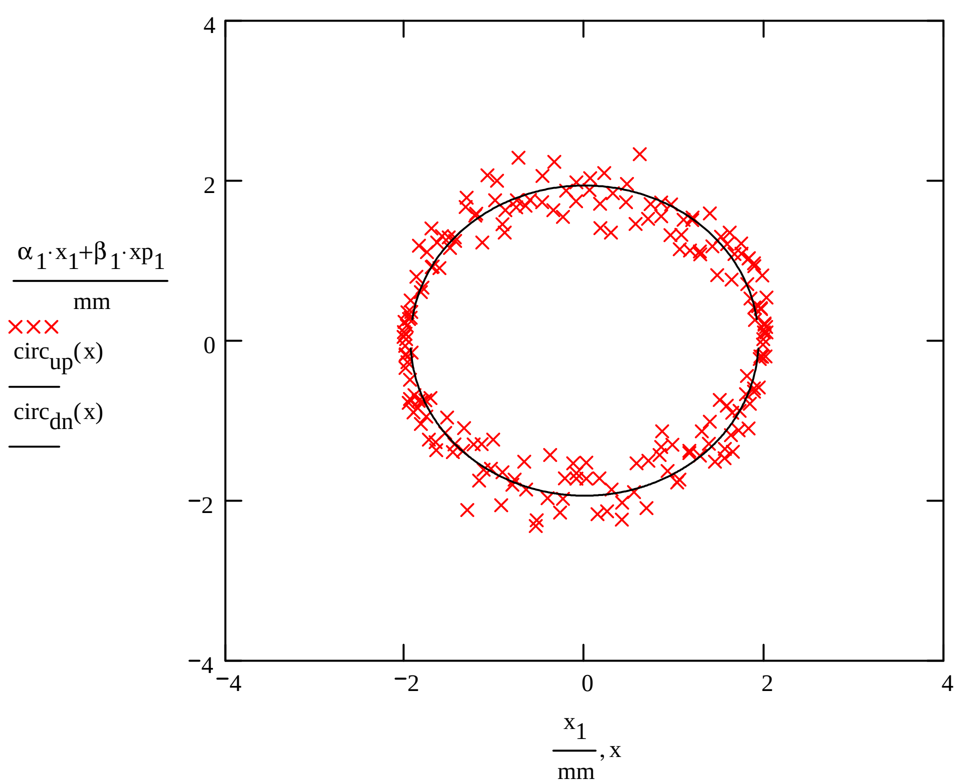
Fit the data to an ellipse: initial guesses: $\alpha_0 := 10$ $\beta_0 := 200 \cdot m$ $a_0 := 10 \cdot mm$

$$\mathrm{Mfcn}(\alpha, \beta, A) := \sum_i \left[\sqrt{\left(x_{1_i}\right)^2 + \left(\alpha \cdot x_{1_i} + \beta \cdot xp_{1_i}\right)^2} - A \right]^2$$

Given $\mathrm{Mfcn}(\alpha_0, \beta_0, a_0) = 0 \cdot m^2$ $\begin{pmatrix} \alpha_1 \\ \beta_1 \\ a \end{pmatrix} := \mathrm{Minerr}(\alpha_0, \beta_0, a_0)$

Results: $\alpha_1 = 19.031$ $\beta_1 = 139.395\,m$ $a = 1.938\,mm$

Plot limits: $amp := 4$ $circ_{up}(x) := \sqrt{\left(\dfrac{a}{mm}\right)^2 - x^2}$ $circ_{dn}(x) := -\sqrt{\left(\dfrac{a}{mm}\right)^2 - x^2}$

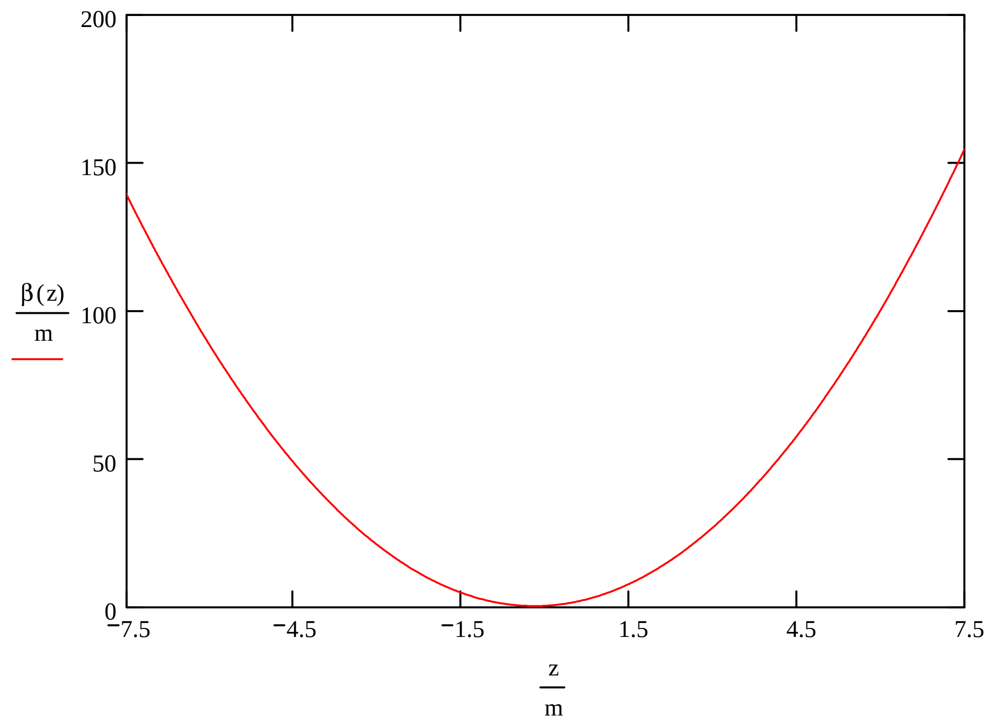


Amplitude Function across the IR:

$$\beta(z) := \beta_1 - 2\cdot\alpha_1\cdot\left(z + L_{star}\right) + \left(\frac{1+\alpha_1^{\,2}}{\beta_1}\right)\cdot\left(z + L_{star}\right)^2$$

$$\alpha(z) := \alpha_1 - \left[\left(\frac{1+\alpha_1^{\,2}}{\beta_1}\right)\cdot\left(z + L_{star}\right)\right] \qquad z_{min} := \alpha_1\cdot\left(\frac{\beta_1}{1+\alpha_1^{\,2}}\right) - L_{star}$$

$$z_{min} = -19.564\,\mathrm{cm}$$



Beta at IP (z = 0 m): $\beta(0\cdot m) = 48.353\,\mathrm{cm}$ $\quad \alpha(0\cdot m) = -0.51$

Beta minimum: $\beta\left(z_{min}\right) = 38.381\,\mathrm{cm}$ $\quad \alpha\left(z_{min}\right) = 0$

Check: ratio of a_2 to a_1: $\sqrt{\dfrac{\beta\left(L_{star}\right)}{\beta\left(-L_{star}\right)}} = 1.053$

Typical oscillation amplitude in the arcs (β = 100 m): $\sqrt{\dfrac{100\cdot m}{\beta_1}}\cdot a = 1.642\,\mathrm{mm}$